Conservation laws.
Generation of physical fields.
Principles of field theories

L.I. Petrova

Moscow State University, Russia, e-mail: ptr@cs.msu.su

Abstract

In the paper the role of conservation laws in evolutionary processes, which proceed in material systems (in material media) and lead to generation of physical fields, is shown using skew-symmetric differential forms.

In present paper the skew-symmetric differential forms on deforming (nondifferentiable) manifolds were used in addition to exterior forms, which have differentiable manifolds as a basis. Such skew-symmetric forms [1], whose existence was established by the author (and which were named evolutionary ones since they possess evolutionary properties), as well as the closed exterior forms, describe the conservation laws. But in contrast to exterior forms, which describe conservation laws for physical fields, the evolutionary forms correspond to conservation laws for material systems.

The evolutionary forms possess an unique peculiarity, namely, the closed exterior forms are obtained from these forms. It is just this that enables one to describe the process of generation of physical fields, to disclose connection between physical fields and material systems and to resolve many problems of existing field theories.

Introduction

Skew-symmetric differential forms possess a peculiarity (which does not possesses any other mathematical apparatus), namely, they describe conservation laws. It is known that closed exterior differential forms are conservative quantities (the differential of closed form vanishes). And closed inexact exterior forms are conservative object: quantities conserved on structures (more precisely, on pseudostructures, which are described by dual forms). Physical structures, from which physical fields are formed and to which the conservation law is assigned, are such conservative objects. From this one can see that closed exterior inexact forms describe conservation laws for physical fields. These are conservation laws for physical fields that can be named exact ones. In field theories by a conception of "conservation laws" is meant just such conservation laws.

However, in physics and mechanics of continuous media the conception of "conservation laws" is related to conservation laws, which can be called balance ones. These are conservation laws for material systems (continuous media) - conservation laws for energy, linear momentum, angular momentum, and mass, which establish a balance between physical quantities and external actions to the system. They are described by differential equations. And in this case from differential equations it follows a relation, which includes a skew-symmetric differential form. This skew-symmetric form [1] possesses the evolutionary properties and from that closed exterior forms corresponding to exact conservation laws are obtained. The passing from the evolutionary form, which correspond to balance conservation laws for material systems, to closed exterior forms, which

1

correspond to conservation laws for physical fields, describes the process of generating physical fields by material systems.

The connection between physical fields and material systems underlines the fact that fields theories describing physical fields have to be based on the principles that specify material system.

1. Conservation laws

It has been noted that in mathematical physics there are two types of conservation laws, namely, conservation laws, which can be called exact ones, and balance conservation laws.

The exact conservation laws are related to physical fields.

The balance conservation laws are conservation laws for material systems (material media). [Material system is a variety of elements that possess internal structure and interact to one another. Thermodynamic and gas dynamical systems, systems of charged particles, cosmic systems, systems of elementary particles and others are examples of material systems. Examples of elements that constitute material system are electrons, protons, neutrons, atoms, fluid particles, cosmic objects and others.]

Below it will be shown that there exists a connection between balance and exact conservation laws which points to a connection between material systems and physical fields.

Exact conservation laws.
The closed exterior differential forms describes exact conservation laws.

From the closure conditions of exterior differential form (vanishing the form differential)

$$d\theta^k = 0 \tag{1}$$

one can see that the closed exterior differential form is a conservative quantity (θ^k is exterior differential form of degree k - (k-form)). This means that it can correspond to conservation law, namely, to some conservative physical quantity.

If the exterior form is a closed inexact form, i.e. is closed only on pseudostructure, the closure condition is written as

$$d_\pi \theta^k = 0 \tag{2}$$

And the pseudostructure π obeys the condition

$$d_\pi {}^*\theta^k = 0 \tag{3}$$

where ${}^*\theta^k$ is a dual form.

From conditions (2) and (3) one can see that the closed exterior form and the dual form constitute a conservative object, namely, a quantity that is conservative on the pseudostructure. Hence, such an object can correspond to some conservation law.

The closure conditions for the exterior differential form ($d_\pi \theta^k = 0$) and the dual form ($d_\pi {}^*\theta^k = 0$) are mathematical expressions of the exact conservation law.

The pseudostructure (dual form) and the conservative quantity (closed exterior form) define a differential-geometrical structure (which is an example of G-Structure). It is evident that such differential-geometrical structure corresponds to exact conservation law.

Below it will be shown that physical structures, which form physical fields, are such differential-geometrical structures.

The mathematical expression for exact conservation law and its connection with physical fields can be schematically written in the following manner:

$$\begin{cases} d_\pi \theta^k = 0 \\ d_\pi {}^*\theta^k = 0 \end{cases} \mapsto \quad \begin{cases} \theta^k \\ {}^*\theta^p \end{cases} \quad - \quad \text{physical structures} \quad \mapsto \quad \text{physical fields}$$

It can be shown that field theories, i.e. theories that describe physical fields, are based on the invariant and metric properties of closed exterior differential and dual forms that correspond to exact conservation laws.

Balance conservation laws.

In mechanics and physics of material systems (of continuous media) the equations of balance conservation laws are used for description of physical quantities, which specify the behavior of material systems. But the balance conservation laws not only define the variation of physical quantities. Their role is much wider. They control evolutionary processes in material systems that are accompanied by an origin of physical structures.

Evolutionary processes are described by the relations that are obtained from the equations of balance conservation laws.

The equations of balance conservation laws are differential (or integral) equations that describe the variation of functions corresponding to physical quantities [2-4]. The functions for equations of material media sought are usually functions which relate to such physical quantities like a particle velocity (of elements), temperature or energy, pressure and density. Since these functions relate to one material system, it has to exist a connection between them. This connection is described by the state-function.

From the equations of balance conservation laws one gets the relation for state-function containing unclosed skew-symmetric differential form. Such a relation, which appears to be nonidentical one (since this relation includes unclosed form), just describes evolutionary processes in material media.

The derivation of this relation can be demonstrated by the example of equations that describe the balance conservation laws for energy and linear momentum.

We introduce two frames of reference: the first is an inertial one (this frame of reference is not connected with material system), and the second is an accompanying one (this system is connected with the manifold built by trajectories of material system elements). The energy equation in inertial frame of reference can be reduced to the form:

$$\frac{D\psi}{Dt} = A_1 \tag{4}$$

3

where D/Dt is the total derivative with respect to time, ψ is the functional of the state that specifies the material system, A_1 is the quantity that depends on specific features of the system and on external energy actions onto the system. {The action functional, entropy, wave function can be regarded as examples of the functional ψ. Thus, the equation for energy presented in terms of the action functional S has a similar form: $DS/Dt = L$, where $\psi = S$, $A_1 = L$ is the Lagrange function. In mechanics of continuous media the equation for energy of ideal gas can be presented in the form [4]: $Ds/Dt = 0$, where s is entropy. }

In the accompanying frame of reference the total derivative with respect to time converts into the derivative along trajectory. Equation (4) is now written in the form

$$\frac{\partial \psi}{\partial \xi^1} = A_1 \tag{5}$$

here ξ^1 is the coordinate along trajectory.

In a similar manner, in the accompanying frame of reference the equation for linear momentum appears to be reduced to the equation of the form

$$\frac{\partial \psi}{\partial \xi^\nu} = A_\nu, \quad \nu = 2, ... \tag{6}$$

where ξ^ν are the coordinates in the direction normal to trajectory, A_ν are the quantities that depend on specific features of the system and external (with respect to local domain) force actions.

Eqs. (5), (6) can be convoluted into the relation

$$d\psi = A_\mu \, d\xi^\mu, \quad (\mu = 1, \nu) \tag{7}$$

where $d\psi$ is the differential expression $d\psi = (\partial \psi / \partial \xi^\mu) d\xi^\mu$ (the summation over repeated indices is implied).

Relation (7) can be written as

$$d\psi = \omega \tag{8}$$

here $\omega = A_\mu \, d\xi^\mu$ is the skew-symmetric differential form of first degree.

Since the equations of balance conservation laws are evolutionary ones, the relation obtained is also an evolutionary relation.

Relation (8) was obtained from the equation of balance conservation laws for energy and linear momentum. In this relation the form ω is that of first degree. If the equations of balance conservation laws for angular momentum be added to the equations for energy and linear momentum, this form in the evolutionary relation will be a form of second degree. And in combination with the equation of balance conservation law of mass this form will be a form of degree 3.

Thus, in general case the evolutionary relation can be written as

$$d\psi = \omega^p \tag{9}$$

where the form degree p takes values $p = 0, 1, 2, 3..$ (The evolutionary relation for $p = 0$ is analogue to that in differential forms, and it was obtained from

4

interaction of energy and time.) In relation (8) the form ψ is a form of zero degree. And in relation (9) the form ψ is a form of $(p-1)$ degree.

Let us show that *the evolutionary relation obtained from the equation of balance conservation laws proves to be nonidentical.*

To do so we shall analyze relation (8).

In the left-hand side of evolutionary relation (8) there is the differential that is a closed form. This form is an invariant object. The right-hand side of relation (8) involves the differential form ω, that is not an invariant object since in real processes, as it will be shown below, this form proves to be unclosed.

For a form to be closed the differential of the form or its commutator must be equal to zero. Let us consider the commutator of the form $\omega = A_\mu d\xi^\mu$. The components of commutator of such a form can be written as follows:

$$K_{\alpha\beta} = \left(\frac{\partial A_\beta}{\partial \xi^\alpha} - \frac{\partial A_\alpha}{\partial \xi^\beta} \right) \tag{10}$$

(here the term connected with the manifold metric form has not yet been taken into account).

The coefficients A_μ of the form ω have been obtained either from the equation of balance conservation law for energy or from that for linear momentum. This means that in the first case the coefficients depend on energetic action and in the second case they depend on force action. In actual processes energetic and force actions have different nature and appear to be inconsistent. The commutator of the form ω constructed from derivatives of such coefficients is nonzero. This means that the differential of the form ω is nonzero as well. Thus, the form ω proves to be unclosed and is not a measurable quantity.

This means that relation (8) involves a noninvariant term. Such a relation cannot be an identical one. Hence, without a knowledge of concrete expression for the form ω, one can argue that for actual processes the relation obtained from the equations corresponding to balance conservation laws proves to be nonidentical.

Similarly it can be shown that general relation (9) is also nonidentical. (The analysis of some particular equations of balance conservation laws and relevant evolutionary relations are presented in papers [1]).

{The peculiarities of nonidentity of evolutionary relation are connected with the differential form ω^p that enters into this relation. The form ω^p in evolutionary relation is a skew-symmetric differential form. However, this form is not *exact* one. Unlike to exterior form, whose basis is a differential manifold, this form is defined on deforming (nondifferentiable) manifold. (About properties of such skew-symmetric form one can read, for example, in paper [1]). The peculiarity of skew-symmetric forms defined on such manifold is the fact that their differential depends on the basis. The commutator of such form includes the term that is connected with a differentiating the basis. This can be demonstrated by the example of a skew-symmetric form of first-degree.

Let us consider the first-degree form $\omega = a_\alpha dx^\alpha$. The differential of this form can be written as $d\omega = K_{\alpha\beta} dx^\alpha dx^\beta$, where $K_{\alpha\beta} = a_{\beta;\alpha} - a_{\alpha;\beta}$ are components of commutator of the form ω, and $a_{\beta;\alpha}$, $a_{\alpha;\beta}$ are covariant derivatives. If we express the covariant derivatives in terms of connectedness (if it is possible), they can be written as $a_{\beta;\alpha} = \partial a_\beta/\partial x^\alpha + \Gamma^\sigma_{\beta\alpha} a_\sigma$, where the first term results from differentiating the form coefficients, and the second term results from differentiating the basis. If we substitute the expressions for covariant derivatives into the formula for commutator components, we obtain the following expression for

5

commutator components of the form ω:

$$K_{\alpha\beta} = \left(\frac{\partial a_\beta}{\partial x^\alpha} - \frac{\partial a_\alpha}{\partial x^\beta}\right) + (\Gamma^\sigma_{\beta\alpha} - \Gamma^\sigma_{\alpha\beta})a_\sigma \tag{11}$$

Here the expressions $(\Gamma^\sigma_{\beta\alpha} - \Gamma^\sigma_{\alpha\beta})$ entered into the second term are just components of commutator of the first-degree metric form that specifies the manifold deformation and hence is nonzero. (In the commutator of exterior form, which is defined on differentiable manifold, the second term absents: the connectednesses are symmetric, that is, the expression $(\Gamma^\sigma_{\beta\alpha} - \Gamma^\sigma_{\alpha\beta})$ vanishes). [It is well-known that the metric form commutators of first-, second- and third degrees specifies, respectively, torsion, rotation and curvature.]

The skew-symmetric form in evolutionary relation is defined on the manifold made up by trajectories of the material system elements. Such a manifold is a deforming manifold. The commutator of skew-symmetric form defined on such manifold includes the metric form commutator being nonzero. (In expression (10) one more term connected with the torsion of accompanying manifold on which the form $\omega = A_\mu d\xi^\mu$) is defined will appear. The commutator of such skew-symmetric form cannot be equal to zero. And this means that evolutionary skew-symmetric form, which enters into evolutionary relation, cannot be closed.

Nonclosure of evolutionary form and the properties of commutator of such form define properties and peculiarities of the relation obtained from the equations of balance conservation laws.}

Below it will be shown that the properties and peculiarities of nonidentical evolutionary relation enables one to understand the mechanism of evolutionary processes in material systems and the mechanism of generation of physical fields.

2. Connection between physical fields and material systems. Generation of physical fields

The nonidentity of evolutionary relation means that the balance conservation law equations are inconsistent (nonconjugated). This reflects the properties of the balance conservation laws that have a governing importance for the evolutionary processes in material media, namely, their *noncommutativity*.

The noncommutativity of balance conservation laws causes the fact that the material system state appears to be nonequilibrium one.

It is evident that, if the balance conservation laws be commutative, the evolutionary relation would be identical and from that it would be possible to get the differential $d\psi$ and find the state-function, and this would indicate that the material system is in equilibrium state.

However, as it has been shown, in real processes the balance conservation laws are noncommutative. The evolutionary relation is not identical and from this relation one cannot get the differential $d\psi$. This means that the system state is nonequilibrium. It is evident that the internal force producing such nonequilibrium state is described by the evolutionary form commutator. Everything that gives contribution to the commutator of the form ω^p leads to emergence of internal force.

Nonidentical evolutionary relation also describes how the state of material system changes. This turns out to be possible due to the fact that the evolutionary nonidentical relation is a selfvarying one. This relation includes two objects one of which appears to be unmeasurable. The variation of any object of the

6

relation in some process leads to variation of another object and, in turn, the variation of the latter leads to variation of the former. Since one of the objects is a unmeasurable quantity, the other cannot be compared with the first one, and hence, the process of mutual variation cannot stop. This process is governed by the evolutionary form commutator, that is, by interaction between the commutator made up by derivatives of the form itself and by metric form commutator of deforming manifold made up by trajectories of elements of material system.

Selfvariation of nonidentical evolutionary relation points to the fact that the nonequilibrium state of material system turns out to be selfvarying. The state of material system changes but holds nonequilibrium during this process.

During selfvariation of evolutionary relation it can be realized conditions when an inexact (closed *on pseudostructure*) exterior form is obtained from evolutionary form. This leads to the fact that from nonidentical evolutionary relation it will be obtained an identical (on pseudostructure) relation, and this points to the transition of material system from nonequilibriun state to locally equilibrium state.

The transition from unclosed evolutionary form to closed exterior form is possible only as degenerate transformation, namely, a transformation that does not conserve the differential. The conditions of degenerate transformation are those that determine the direction on which interior (only along a given direction) differential of evolutionary form vanishes. These are conditions that define the pseudostructure, i.e. the closure conditions of dual form, and lead to realization of the exterior form closed on pseudostructure. [The conditions of degenerate transformation are some symmetries. Such conditions can be due to degrees of freedom of material system (like, for example, translation, rotation, oscillation and so on) that are realized while selfvarying of nonequilibrium state of material system.]

As it has been already mentioned, the differential of the evolutionary form ω^p involved into nonidentical relation (9) is nonzero. That is, $d\omega^p \neq 0$. If the conditions of degenerate transformation are realized, it will take place the transition

$d\omega^p \neq 0 \rightarrow$ (degenerate transformation) $\rightarrow d_\pi \omega^p = 0$, $d_\pi{}^* \omega^p = 0$

The relations obtained

$$d_\pi \omega^p = 0, d_\pi{}^* \omega^p = 0 \tag{12}$$

are closure conditions for exterior inexact form and for dual form. This means that it is realized an exterior form closed on pseudostructure.

In this case, on the pseudostructure π evolutionary relation (9) converts into the relation

$$d_\pi \psi = \omega_\pi^p \tag{13}$$

which proves to be an identical relation. Since the form ω_π^p is a closed one, on the pseudostructure this form turns out to be a differential. There are differentials in the left-hand and right-hand sides of this relation. This means that the relation obtained is an identical one.

From identical relation one can obtain the state differential and find the state function, and this points to the material system state is a equilibrium state. But this state is realized only locally since the state differential is interior one defined exclusively on pseudostructure. (*The total state of material system turns out to be nonequilibrium* because the evolutionary relation itself remains to be nonidentical one.)

Relation (13) holds the duality. The left-hand side of relation (13) includes the differential, which specifies material system and whose availability points to the locally-equilibrium state of material system. And the right-hand side includes the closed inexact form, which is a characteristics of physical fields. The closure conditions (12) for exterior inexact form correspond to conservation law, i.e. to a conservative on pseudostructure quantity, and describe a differential-geometrical structure. These are such structures (pseudostructures with conservative quantities) that are physical structures formatting physical fields. Massless particles, charges, structures made up by eikonal surfaces and wave fronts, and so on are examples of physical structures.

The transition from nonidentical relation (9) obtained from balance conservation laws to identical relation (13) means the following. Firstly, the existence of state differential (left-hand side of relation (13)) points to transition of material system from nonequilibrium state to locally-equilibrium state. And, secondly, the emergence of closed (on pseudostructure) inexact exterior form (right-hand side of relation (13)) points to origination of physical structure (from which physical fields are made up).

The duality of identical relation also explains the duality of nonidentical evolutionary relation. On the one hand, the evolutionary relation describes the evolutionary process in material systems, and, on the other, describes the process of emergence of physical structures and generating physical fields.

The emergence of physical structures in evolutionary process reveals in material system as an advent of certain observable formations, which develop spontaneously. Such formations and their manifestations are fluctuations, turbulent pulsations, waves, vortices, and others. It appears that structures of physical fields and the formations of material systems observed are a manifestation of the same phenomena. The light is an example of such a duality. The light manifests itself in the form of a massless particle (photon) and of a wave.

By sequential integrating of the evolutionary relation the closed inexact exterior forms of degree k are obtained from the evolutionary form of degree p, where k ranges from p to 0. In this case the pseudostructures of dimensions $(n + 1 - k)$ correspond to closed forms of degree $k = p$, $k = p - 1$, ..., $k = 0$. {Under degenerate transformation from the nonidentical evolutionary relation one obtains a relation being identical on pseudostructure, that can be integrated. The relation obtained after integration proves to be nonidentical as well. By sequential integrating the nonidentical relation of degree p (in the case of realization conditions of corresponding degenerate transformations and forming the identical relation), one can get a closed (on the pseudostructure) exterior forms of appropriate degrees.}

The parameters of evolutionary and exterior forms p, k, n enables one to introduce the classification of physical structures that defines a type of physical

structures and, accordingly, of physical fields and interactions (See, Appendix).

Since the physical structures are generated by material media, their characteristics are specified by characteristics of material systems, by the characteristics of evolutionary form and of closed exterior form realized and by the quantity of nonvanishing commutator of evolutionary form [1]. (Specifically, the closed exterior form realized defines such a characteristics like a charge).

In conclusion of this section it should be emphasized the role of conservation laws in generation of physical fields.

The nonidentity of evolutionary relation obtained from the equations that describe conservation laws for material systems (material media) points to a noncommutativity of these conservation laws, which are balance ones rather then exact. The noncommutativity of conservation laws leads to evolutionary processes in material media, which gives rise to generation of physical fields. The generation of physical fields is caused by the fact that due to availability of material system degrees of freedom the conditions, under which the balance conservation laws locally (only under these conditions) commutate and become an exact conservation laws, are realized in material system. And this points to emergence of physical structures from which physical fields are formed.

The connection between physical fields and material systems has to be taken into account in field theories as well.

3. Basic principles of existing field theories

It can be shown that the field theories are based on invariant and metric properties of closed exterior (inexact) differential and dual forms, which correspond to exact conservation laws.

The properties of closed exterior and dual forms, namely, invariance, covariance, conjugacy, and duality, lie at the basis of the group, structural and other invariant methods of field theories.

The nondegenerate transformations of field theory are transformations of closed exterior form - nondegenerate transformations conserving the differential.

These are gauge transformations for spinor, scalar, vector, and tensor fields, which are transformations of closed (0-form), (1-form), (2-form) and (3-form) respectively.

The gauge, i.e. internal, symmetries of field theory (corresponding to gauge transformations) are those of closed exterior forms. The external symmetries of the equations of field theory are symmetries of closed dual forms.

The field theory operators are connected with nondegenerate transformations of exterior differential forms [5].

It can be shown that the equations of existing field theories are those obtained on the basis of the properties of exterior form theory.

In equations of existing field theories the closure conditions of exterior or dual forms are laid. The postulates on which the equations of existing field

9

theories are such conditions. Closed inexact or dual forms are solutions of the field-theory equations.

The Hamilton formalism is based on the properties of closed exterior form of the first degree and corresponding dual form. From the set of Hamilton equations and from corresponding field equation the identical relation with exterior form of first degree, namely, the Poincare invariant $ds = -H\,dt + p_j\,dq_j$ is obtained.

The Schrödinger equation in quantum mechanics is an analog to field equation, where the conjugated coordinates are replaced by operators connected with the exterior forms of zero degree. The Heisenberg equation corresponds to the closure condition of dual form of zero degree. Dirac's *bra-* and *cket-* vectors made up a closed exterior form of zero degree. It is evident that the relations with closed skew-symmetric differential and dual forms of zero degree correspond to quantum mechanics.

The properties of closed exterior form of second degree (and dual form) lie at the basis of the electromagnetic field equations. The strength tensor $F_{\mu\nu}$ in the Maxwell equations obeys the identical relations $d\theta^2 = 0$, $d^*\theta^2 = 0$ [5], where $\theta^2 = \frac{1}{2}F_{\mu\nu}dx^\mu dx^\nu$ is a closed exterior form of second degree.

Closed exterior and dual forms of third degree correspond to gravitational field. The Einstein equation is a relation in differential forms that relates the differential of dual form of first degree (Einstein's tensor) and the closed form of second degree – the energy-momentum tensor. And it can be shown that Einstein's equation is obtained from the relations which connect the differential forms of third degree [6].

One can recognize that equations of field theories, as well as the gauge transformations and symmetries, are connected with closed exterior forms of given degree. This enables one to introduce a classification of physical fields and interactions according to the degree of closed exterior form. This shows that there exists a commonness between field theories describing physical fields of different types. The degree of closed exterior forms is a parameter that integrates fields theories into unified field theory.

Thus, it is evident that field theories are based on the properties of closed exterior and dual forms.

However, in existing field theories there are no answers to following questions.

1. From what one may take closed exterior forms that correspond to conservation laws and on which properties field theories are based?

2. What defines the degree of closed exterior forms that can be a parameter of unified field theory? Why this parameter varies from 0 to 3?

3. By what the quantum character of field theories is conditioned?

4. By what the symmetries and transformations of field theories are conditioned?

The evolutionary skew-symmetric forms enable one to answer these questions.

It was shown that the evolutionary forms allow to describe the process of generation of physical fields, which discloses a connection between physical fields and material systems. And this points to the fact that at the basis of field

theories, i.e. theories that describe physical fields, it has to lie the principles taking into account the connection of physical fields and material systems.

4. On foundations of field theory

In the second section it had been shown that closed exterior forms, which correspond to conservation laws for physical fields and on which properties the theories describing physical fields are based, are connected with the equations for material systems. These closed exterior forms are obtained from evolutionary forms in nonidentical relation derived from the equations of balance conservation laws for material systems.

And it was shown that the degrees of relevant closed forms are connected with the degree p of evolutionary form in the nonidentical relation. (It should be recalled that the degree of evolutionary form p is connected with the number of interacting balance conservation laws for material media and can take the values $0, 1, 2, 3$. In this case from the nonidentical relation with evolutionary form of degree p the closed (inexact) forms of degrees k, which can take the values $p, p-1, ..., 0$, are obtained in the process of sequential integrating (if the degenerate transformations are realized).)

Since physical fields, as it had been shown, are formed up by physical structures, this means that physical fields are discrete ones rather then continuous. The exterior closed forms corresponding to conservation laws are *inexact* forms because they are obtained only under degenerate transformations. Hence, the conservation laws corresponding to physical fields are satisfied on physical structures only. (For physical fields be continuous ones, the exact exterior forms must correspond to these fields).

The discreteness of physical fields points to the fact that field theories mast be quantum ones.

By what symmetries and transformations of field theories are conditioned?

The external symmetries of the equations of field theory are symmetries of closed dual forms. It had been shown that the symmetries of dual forms are connected with the condition of degenerate transformations, which are realized in the process of selfvariation of material system. It is clear that such symmetries are conditioned by degrees of freedom of material system (translational, rotational, oscillatory and so on). Hence, the external symmetries of the equations of field theory are also conditioned by degrees of freedom of material system.

The gauge, i.e. internal, symmetries of the field theory (corresponding to the gauge transformations) are those of closed exterior forms. The symmetries of closed exterior forms are symmetries of differentials of skew-symmetric forms (the closure conditions of the form, namely, vanishing the form differential, are connected with these forms). The differential of closed inexact form obtained from evolutionary form (and corresponding to physical structure) is an interior, being equal to zero, differential of evolutionary form. This differential is obtained from the evolutionary form coefficients and therefore is connected with the characteristics of material system. As the result, the symmetries of closed exterior forms, and, consequently, the interior symmetries of field theory, are defined by the characteristics of material system.

The symmetry of dual forms lead to degenerate transformations, i.e. to going from evolutionary forms (with nonzero differential) to closed exterior forms (with the differential being equal to zero). And the symmetries of exterior forms lead to nondegenerate transformations, namely, to transitions from one closed form to another closed form. Thus, it appears that degenerate and nondegenerate transformations are interrelated.

This is also valid for transformations in field theory, since the interior and exterior symmetries of field theories are connected with the symmetries of closed exterior and dual forms. Thus, it turns out that the gauge nondegenerate transformations of field theories are connected with degenerate transformations. The transformations of field-theory equations, to which the exterior symmetries correspond, are such degenerate transformations.

What are general properties of the equations of field theories?

The equations of fields theories, which describe physical fields, must be connected with the equations that describe material systems, since material systems generate physical fields.

The equations of field theory are equations for functionals like wave-function, action functional, entropy and so on.

The equations of material systems are partial differential equations for desired functions like a velocity of particles (elements), temperature, pressure and density, which correspond to physical quantities of material systems (continuous media). It had been shown that from such equations it is obtained the evolutionary relation for functionals (and state-functions) like wave-function, action functional, entropy and others, in other words, for functional of field theories. And this points to the fact that the field-theory equations must be connected with the evolutionary relation derived from the equations for material systems.

If the nonidentical evolutionary relation be regarded as the equation for deriving identical relation with include closed forms (describing physical structures desired), one can see that there is a correspondence between such evolutionary relation and the equations for functional of existing field theories. It can be verified that the equations of existing field theories are either such equation or is analogous (differential or tensor) to such equation. The solutions of field-theory equations are identical relation obtained from nonidentical evolutionary relation.

The results obtained show that when building the general field theory it is necessary to take into account the connection of existing field theories (which are based on the conservation laws for physical fields) with the equations of noncommutative conservation laws for material media (the balance conservation laws for energy, linear momentum, angular momentum and mass and the analog to such laws for the time, which takes into account a noncommutativity of time and energy of material system).

The theories of exterior and evolutionary skew-symmetric differential forms, which reflect the properties of conservation laws for physical fields and material media, allow to disclose and justify the general principles of field theories and may serve as an approach to general field theory.

Appendix

Below we present the table where physical fields and interactions in their dependence on the parameters p, k, n of evolutionary and closed exterior forms are demonstrated. (Here p is the degree of evolutionary form in nonidentical relation, which is connected with the number of interacting balance conservation laws, k is the degree of closed form generated by nonidentical relation and n is the dimension of original inertial space.)

This table corresponds to elementary particles.

[It should be emphasized the following. Here the concept of "interaction" is used in a twofold meaning: the interaction of balance conservation laws that relates to material systems, and the physical concept of "interaction" that relates to physical fields and reflects interactions of physical structures, namely, it is connected with exact conservation laws].

TABLE

interaction	$k \backslash p, n$	0	1	2	3
gravitation	3				**graviton** ⇑ electron proton neutron photon
electro-magnetic	2			**photon2** ⇑ electron proton neutrino	**photon3**
weak	1		**neutrino1** ⇑ electron quanta	**neutrino2**	**neutrino3**
strong	0	**quanta0** ⇑ quarks?	**quanta1**	**quanta2**	**quanta3**
particles material nucleons?	exact forms	electron	proton	neutron	deuteron?
N		1 time	2 time+ 1 coord.	3 time+ 2 coord.	4 time+ 3 coord.

In the Table the names of the particles created are given. Numbers placed near particle names correspond to the space dimension. Under the names of particles the sources of interactions are presented. In the next to the last row we present particles with mass (the elements of material system) formed by interactions (the exact forms of zero degree obtained by sequential integrating the evolutionary relations with evolutionary forms of degree p corresponding to these particles). In the bottom row the dimension of the *metric* structure created is presented.

From the Table one can see the correspondence between the degree k of closed forms realized and the type of interactions. Thus, $k = 0$ corresponds to strong interaction, $k = 1$ corresponds to weak interaction, $k = 2$ corresponds to electromagnetic interaction, and $k = 3$ corresponds to gravitational interaction.

The degree k of closed forms realized and the number p connected with the number of interacting balance conservation laws determine the type of interactions and the type of particles created. The properties of particles are governed

by the space dimension. The last property is connected with the fact that closed forms of equal degrees k, but obtained from the evolutionary relations acting in spaces of different dimensions n, are distinctive because they are defined on pseudostructures of different dimensions (the dimension of pseudostructure $(n + 1 - k)$ depends on the dimension of initial space n). For this reason the realized physical structures with closed forms of degrees k are distinctive in their properties.

References

1. Petrova L. I. Skew-symmetric differential forms: Conservation laws. Foundations of field theories. -Moscow, URSS, 2006, 158 p. (in Russian).

2. Tolman R. C., Relativity, Thermodynamics, and Cosmology. Clarendon Press, Oxford, UK, 1969.

3. Fock V. A., Theory of space, time, and gravitation. -Moscow, Tech. Theor. Lit., 1955 (in Russian).

4. Clark J. F., Machesney M., The Dynamics of Real Gases. Butterworths, London, 1964.

5. Wheeler J. A., Neutrino, Gravitation and Geometry. Bologna, 1960.

6. Tonnelat M.-A., Les principles de la theorie electromagnetique et la relativite. Masson, Paris, 1959.